Enrollment No. _____________

Shree Manibhai Virani and Smt. Navalben Virani Science College (Autonomous), Rajkot

Affiliated to Saurashtra University, Rajkot

SEMESTER END EXAMINATION NOVEMBER - 2017

M.Sc. Mathematics

16PMTCC15 – LINEAR ALGEBRA

Duration of Exam – 3 hrs ***Semester – III*** ***Max. Marks – 70***

***Part A** (5X2= 10 marks)*

Answer **ALL** questions

1. Define algebra.
2. Define unitary transformation.
3. Define Harmitian Adjoint.
4. Define Trace and Transpose.
5. State Sylvester's law of inertia.

***Part B** (5X5= 25 marks)*

Answer **ALL** questions

6a. For $T \in A(V)$, T is regular if and only if T maps V onto V.

OR

6b. Show that $(\lambda T)^* = \bar{\lambda} T^*$.

7a. State and prove Cayley – Hamilton Theorem

OR

7b. T is unitary if and only if $T^*T = 1$.

8a. If $\mathrm{T} \in \mathrm{A(V)}$ is nilpotent, then $\alpha_0 + \alpha_1 \mathrm{T} + \alpha_2 \mathrm{T}^2 + \cdots + \alpha_m \mathrm{T}^m$ is invertible if $\alpha_0 \neq 0$.

OR

8b. If T is Hamiltonian then all characteristic roots of T are real.

9a. For matrices A and B , show that $tr(AB) = tr(BA)$

OR

9b. For linear transformation S, if $S^*S(v) = 0$ then $S(v) = 0$.

10a. For F of characteristic 0 and $S, T \in A_F(V)$ if $ST - TS$ is commutative with S then $ST - TS$ is nilpotent.

OR

10b. Show that $f: R^3 \times R^3 \to R$ defined by
$f(x,y) = 4x_2y_1 - 4x_1y_2 - x_1y_3 + x_3y_1 - 6x_2y_3 + 6x_3y_2$ is a skew symmetric bilinear form where $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$.

Part C ***(5X7= 35 marks)***

Answer **ALL** questions

11a. Find the inverse of $T \in R^3$, $T(x,y,z) = (x + y + z, 2x + 2y, 3z)$

OR

11b. For linear transformation T defined on finite dimension vector space V, T is singular if and only if there exist non zero vector $v \in V$ such that $T(v) = 0$.

12a. Find the characteristic roots of $T \in R^3$, $T(x,y,z) = (x - y, y - z, z - x)$.

OR

12b For normal transformation N, u and v are characteristic vectors corresponding to distinct characteristic roots λ and μ then $\langle u, v\rangle = 0$.

13a. Using Cramer's rule solve:
$3x + 9y + 4z = 16, -x - y + 10z = 8, 2x + 2y + 3z = 7$

OR

13b. For $T, S \in A(V)$, if S is regular then T and STS^{-1} have the same minimal polynomial.

14a. Find $v \neq 0$, for $T \in R^3$, $T(x,y,z) = (x - 2y + 3z, 7x + 4y - 8z, 6x + 6y - 11z)$ such that $T(v) = 0$.

OR

14b. If $\langle T(u), T(u)\rangle = \langle u, u\rangle$ for all $u \in V$ then T is unitary.

15a. Prove that $Hom(V,V)$ is a group under the operation addition.

OR

15b. Show that $f: R^2 \times R^2 \to R$ defined by $f(x,y) = x_1y_1 + 4x_2y_1 - 3x_1y_2 - x_2y_2$ is a bilinear form where $x = (x_1, x_2)$ and $y = (y_1, y_2)$.